Metalingua – a Formal Language for Integration of Disciplines via their Universes of Discourse

IOACHIM DRUGUS
Associate Researcher of Moldova Academy of Sciences, Chisinau, MOLDOVA

Key words:*disciplines, semantic web, formal languages, logic, knowledge*

Abstract: While the languages of classical logic express truth, *metalingua* is meant to express *sense*. This language is based on three abstractions - *atomification, association* and *aggregation*, which are regarded as fundamental, and due to this, in my previous publications, I have used a symbolic name for it, *A3 language*. This language offers a framework for describing artificial and natural languages and turned out to have an expressive power higher than the language of the logic of predicates. Therefore, in this article I gave it a full-fledged name - *metalingua*.

A strict discipline uses mathematical structures which are defined by mathematicians in terms of set theory and mathematical logic. The first domain of human practice which employs this situation is the next generation of web, the *Semantic Web,* with software, which simulates human understanding and stores knowledge in global knowledge bases. But the formal languages for knowledge representation are very complex for developers and are practically "unreadable" by non-technical people. The objective of my research was to find a simple knowledge representation language based on set theory. This required a revision of set theory and resulted in a theory of universes called *universics* - the language of universics turned out to be metalingua.

In a discipline, we are using structures from the discipline's *universe of discourse*. Since all strict disciplines operate with mathematical structures, their universes of discourse are the same and coincide with the universe of discourse of universics. What makes one discipline different from another is the *sense* attributed to the structures within the universe of discourse. Due to this, metalingua can serve as a language for integration of disciplines by proceeding from their universes of discourse, all of which are part of the universe of discourse of universics.

1. Brain mathematics

Initially, I have developed *metalingua* as a symbolic language for denotation of the fundamental operations of mind. While *artificial intelligence* is preoccupied with study and modelling of intelligence, *brain informatics* is focused on study of the computing aspect of brain. I have introduced the syntagma *brain mathematics* for the research focused on mathematical operations presumably done by the brain as a physical carrier of intelligence (Drugus 2007). Thus, *brain mathematics* is supposed to denote the sub-domain of brain informatics, focused on the fundamental mathematical operations done by the intellect, where by "fundamental" I refer to those operations through which all other operations can be expressed.

A minimal set of mathematical operations irreducible to each other, but though which any operation of a larger class C of operations can be expressed, is said to be an *orthogonal basis* of the class C. As I have substantiated in Drugus 2007, there are exactly three operations – *aggregation, association and atomification* – which make up the orthogonal basis of the class of operations done by the brain. The specialization of brain compartments on these operations (Drugus 2009 a) is shown in Figure 1.

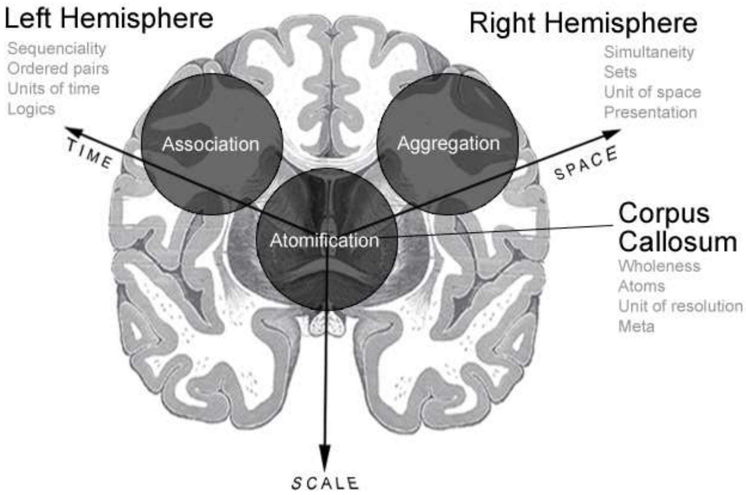

Figure 1. Brain mathematics

By *abstraction* we usually refer to both an operation of the intellect and the result of such an operation. In practice we need not distinguish between these two notions, because in order to use an abstraction we do not need to know how it was obtained by the mind. In a more general approach, like the one of this paper, we have to be more discriminative, and I refer to the first one as *abstraction operation* or just *operation*, and to the second one as *concept*. The Latin etymology (*con+cipere*, “with”+“capturing”) both supports this choice of “concept” for denoting the result of an abstraction operation and describes such an operation as “capturing”, i.e. “extraction” of the concept from the “mind content”. In study of the mechanism of mind, we must treat any abstraction operation as a one-argument operation of extraction of a concept from the *mind content*. But in this paper we are focused on concepts rather than on the mechanism of mind which produces them and I consider the abstraction operations as many-argument mathematical operations defined over and resulting in *entities*.

Same as the physical matter can be regarded as “content” of the Universe, so I regard the Mind as having content and I refer to it by *mind content* (Drugus 2009b). Moreover, I regard both the mind and the universe as *worlds,* where by a *world* I mean a universe together with its content. All these notions are formulated in mathematical terms of *universics* in [Drugus 2009a, 2009b]. The figure 2 illustrates my vision on mind content regarded as a conceptual network “weaved” by the intellect by repeated application of three operations mentioned above.

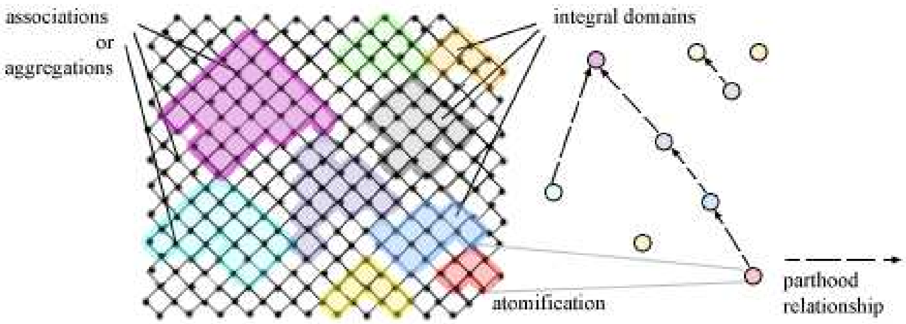

Figure 2. Mind content

The figure above illustrates the mind content obtained in result of a synthesis process, which is directed - in this figure, we consider that the mind content „grows“ upwards. The results of applications of both the aggregation and association operations are represented as nodes of the network. In order to simplify the picture, the distinction between the entity type (*aggregation* or *association*) is not indicated in the figure and all the aggregations in this figure are two-element aggregations – same number of arguments as of the association operation. The colored areas are said to be *integral domains*. These pieces of mind content are created by applying the atomification operation, which can be thought to act in a dimension, different from the dimensions where act association and aggregation. The results of applications of the atomification operation can be imagined to lie in space above the 2D space of the picture. This is why, to represent the integral domains as new entities lying outside the plane and to be able to discriminate between them, they are colored in different colors. The „botom“ of each integral domain is represented by the *atoms* (indivisible constituents) of the represented object. This „botom“ is „dented“ because in the synthesis process the representations of the atoms appear at different times. The top of each integral domain is represented by exactly one entity which we regard as the *unity* of this integral domain.

The first intuitive idea about the manner how an integral domain can be extracted from the mind content is by imagining that such a domain is "carved out", or "captured" from the mind content by taking into account the two boundaries. Namely, the *unity* serves as the top boundary and the atoms as the bottom boundary of a *unit* of mind content representing something treated as a "whole".

2. Three basic concepts of the intellect

We are now changing the focus from the manner how the concepts are obtained to the concepts themselves and, therefore, we will re-use the names of the abstraction operations for the concepts obtained in result of such operations. My vision called *universics* posits three abstraction operations and three concepts in the basis of the mechanism of mind. Each of them can be subject matter of a separate discipline. Such is *aggregation*, which is (partially) studied by axiomatic *set theories* or *theory of classes*, in terms of which, by using mathematical logics, are formulated all mathematical notions.

The abstraction of *association* is same wealthy in manifestations and can compete for the position of an alternative foundational framework of mathematics, but it did not get an attention of mathematicians comparable with the aggregation. Only lately the association caught the attention of Semantic Web developers who, though, did not give it a name to it and use it unconsciously while representing knowledge via oriented graphs labelled by reserved words.

The atomification operation seems to have been almost completely ignored by mathematicians and got some amount of attention only in formal languages studied in computer science, but it can also serve as an alternative foundational framework for mathematics.

Even though each of the 3 abstractions can serve as a foundational framework for whole mathematics, this is possible only due to modelling the other abstractions via the abstraction posited in foundation. Modelling provides a good basis for reasoning but a model is always different from the modelled phenomenon and the model necessarily ignores some information which might be essential. Therefore, even though an abstraction operation can be modelled via the other two ones, as mathematical operations, the 3 abstraction operation are independent on each other and we will treat them on equal basis.

Universics can be treated as a theory and as such we will consider that the universe of discourse of this theory is "populated" by all the entities obtained in result of repeated application of the three abstraction operations. I said such entities to be universes (Drugus 2009a). Below, we will focus on each of these three concepts in more detail.

2.1. Aggregations

The first concept on which we will focus is the abstraction of *aggregation*. The correct intuition for this concept is provided by the etymology (*ag-greg*, "towards flock") which suggests that by "aggregating" we bring many things together into one "flock", i.e. into something which is governed by no order. The notion of *aggregation*, *class* and *bag* used in IT, as well as the notion of *container* and *collection* used in Semantic Web, are examples of *aggregation* conforming with the intuition of French "togetherness" or of the ancient Greek "flock".

The concept of *aggregation* was the first concept to be formalized by mathematics. It obtained a mathematical counterpart as a strict notion of *set* and later as its generalization to the notion of *class*. True, in mathematics there is no widely accepted name for the abstraction operation and we will use the term *aggregation* to refer both to the abstraction operation for creating a set in set theory or a class in class theory.

The notion of *set* was introduced in the 19^{th} century in mathematical practice by Georg Cantor who used for this notion the French word *ensemble* ("together"). This suggests that he meant to formalize by the notion of *set* exactly the intuitive idea of "togetherness" or, in our terms – of *aggregation*. The denotation of a finite set by the list of its elements enclosed within braces $\{a_1,\dots, a_n\}$ is a well-known and widely used denotation and it will be used also for denotation of a finite aggregation.

All axiomatisations of the notion of set, which were introduced later, contain the "extensionality axiom", and moreover - usually include it in the list of axioms as the first axiom. The *extensionality axiom* states: "for any sets A and B, A=B if and only if for any entity x, $x \in A$ if and only if $x \in B$". But there is no intuitive reason to consider equal two aggegations with same members and the natural language supports this by expressions like "two aggregations" in formulation of this idea. Therefore, to keep the view most general, I will not request the arbitrary aggregations to satisfy the extensionality axiom.

Since the new concept is more general than the notion of *set* and even more general than the notion of *class*, it requires a new name and I will use for it the name of the abstraction operation whereby it is obtained – *aggregation*. I will treat extensionality axiom as a constraint on the number of *identities* which a class can have – this axiom states that a class (in particular, a set) has exactly one identity, and this identity is determined by the elements of the class. Based on this, we can simplify our discourse by considering an aggregation to be a class together with an entity called *identity* of the class. Thus, aggregations can be treated as *multi-identity* classes. Due to this, each time when we specify an aggregation, if we are not interested in its specific identity defined in the process of specification, we can refer to the aggregation as just *class*. On the other hand, we can consider the notion of class as an abstraction of higher level – an abstraction obtained from aggregations by ignoring their multiple identities. Conversely, we can say a class to be an un-identified aggregation.

I regard aggregation as a specialty of the right hemisphere of brain for right-handed people – the hemisphere also specialized on processing simultaneity and space (Drugus 2007).

2.2. Associations

By a*ssociation* I refer to the abstraction operation for creating an ordered pair and to any concrete example of the result of application of this operation. I am treating the association abstraction as a specialty of the left hemisphere for the right-handed people – a hemisphere also specialized on processing sequentiality and time.

I denote an association of two entities *a* and *b*, in this order, by the (*a* : *b*) – a denotation which differs from the denotation (*a*, *b*) of an ordered pair used in set theory. The different manner of denotation is justified by the fact that I am treating comma as an operator with properties (like commutativity, associativity, idemponence) different from those expected from a separator for association. Therefore, the separator sign (:) widely used in many domains, including IT, is a better sign for denotation of an association.

In set theory the notion of ordered pair is defined in terms of sets and, probably, the definition which is best known is that of Kuratowsky: (*a*, *b*) = {{a}, {a, b}}. Such a definition is good only for theory, but for an agent viewing the universe of sets and ordered pairs defined as a special type of sets, this definition creates problems. Namely, if an agent views the set {{*a*}, {*a*, *b*}}, he does not know, whether to consider this an ordered pair or a set which, coincidentally, has this form. Also, an author who meant {{*a*}, {*a*, *b*}} to be a set, cannot encrypt this information in this denotation and would have to add it in words like „where by the {{*a*}, {*a*, *b*}} I mean a set and not an ordered pair". For a virtual machine there can appear also other problems due to incomplete encryption of information. Finally, there are infinitely many modes to represent an ordered pair as a set. For all these and other reasons, I regard the notion of ordered pair as *orthogonal* to the notion of set.

An association can be treated as an identified ordered pair and conversely, an ordered pair can be regarded as an abstraction of higher level which neglects different identities of the same ordered pair.

2.3 Atomifications

The notion of *atom* as an abstract concept (and not as a common name for physical atoms) was first used in set theory to refer to a non-set or used in class theory to refer to a non-class. Since we have allowed association as a concept orthogonal to the notion of aggregation, we now have to give a more precise definition.

I consider *structure* an entity built by multiple application of aggregation and association and *atom* – as a non-structure. This treatment implies that the mind is capable of doing, alongside aggregation and association, yet another abstraction operation for creating an atom out of a structure. I say this operation to be *atomification*.

Atomification does not imply any actions upon the structure – it implies changing the view of the agent. Namely, atomification is neglecting any structure of the viewed entity and regarding it as a whole. Due to this intuition, an "atom" should be regarded as a "whole", and we can chose between these two words for a term to denote the same concept. I have chosen the word "atom" because it is easier in use than than the word "whole" (say, "atom" has a plural, but "whole" does not).

Notice that the result of atomification, an atom, can serve as a "building block" for another structure. This shows that the notion of atom and the notion of whole are really relative to each other – if the entity A is regarded as an atom in a structure S due to our making abstraction from the structure S8 of the entity A, same A is a whole in the structure S'. This situation is well illustrated by the integral domains, which represent structures, in the Figure 2.

3. Definition of metalingua

Metalingua is a simple formal language which uses regular words of a natural language called main symbols, the sign of equality, 3 reserved words - **aggregation**, **association** and **atomification**, and 3 types of brackets for building compound expressions of the language, namely:

1. Braces "{" and "}", which enclose a list, the elements of which are separated by "," (comma);
2. Parentheses "(" and ")", which enclose a list, the elements of which are separated by ":" (colon);
3. Round brackets "[" and "]" which enclose an expression of the language.

This simple language can be used for denotation of the entities in set theory or, wider – of the entities of universics, and maybe in less strict symbolics, it has been used for a century and a half since set theory came into being. But this language obtains a completely different use if we widen the meaning of this symbolics. In order to describe the full semantics of this language we will go through an analysis of what is *sense* and will offer a mathematical treatment to this notion. Meanwhile, I will introduce the semantics of metalingua on the intuitive level.

In denotation, we proceed from an entity and end up with a notation, in discourse (a generally accepted term for "discussion") we proceed conversely – we start with a notation and after the interpretation we obtainan entity. But there can exist many interpretations and, therefore, this inverse direction brings up a whole class of entities denoted by the same notation - the interpretations are limited only by the sense of notations. Thus, in discourse, all the denotations of metalingua obtain a meaning different from the meaning we discussed earlier.

In denotation, the braces serve for writing down the notation of a set (aggregation). In discourse will treat the braces as specification of a *correlation*. While the notion of *relationship* with a given list of *correlates* is widely used in mathematics and has a formal counterpart in the language of set theory, the word "correlation" is used informally. In metalingua the notion of *correlation* obtains a strict sense of "relationship with named correlates", which we will explain by two examples below.

The comparison two numbers *m* and *n* denoted by "*m* < n" is said to be a relationship. Which of the two numbers is smaller and which is larger depends on their place in this denotation and we cannot change the two numbers with places and still keep the same relationship between them. On the contrary, if we denote the same fact this manner: {*smaller* : m, *larger* : n}, then we can safely change the places of the two elements of the list enclosed between braces. We will say this second type of notation to be a *correlation*, and for the concrete example above – a correlation with the correlates "smaller" and "larger". Another example of a correlation is the correlation we talked above: {metro-language, meta-language}. Notice, that here we indicated a correlation as an abstraction, while in the numeric example earlier (with colons), we indicated a concrete manifestation of a correlation.

In denotation process, (*a* : *b*) denotes an ordered pair of entities. In discourse this becomes a complex operation of which we will say to be *modification* of *b* by *a*. "Modifier" is the common name used in linguistics for an adjective, adverb or complement – all these "modify" the sense of a word or expression to which they apply. For example, by applying the modifier "white" to the word "horse" we obtain "white horse", which in symbolic notation should be written like this - (white : horse). Here the use of parentheses is important and are regarded as part of the formal language.

In a discourse, the square brackets are treated a manner similar to the quotation marks – they enclose the denotation of the entity to which we refer or the entity itself. For example, to apply the meaning of "white horse" to a horse named "Derby" we would formally write down the expression - ((white : horse): [Derby])

4. Sense and reference

The two papers [Frege 1892] and [Russel 1905] stood in the beginning of research of *sense* and *reference* – a beginning which unfortunately did not get a continuation. The notion of reference obtained a key importance for Semantic Web only lately but, as to my knowledge, the notion of sense did not get a widely accepted formalization up to current days and remains an obscure and arcane domain.

I say an entity *e* to be a *reference* in language L to an entity *v*, if *e* represents *v* in L, or more precisely, if *e* is a *representative* in L of *v* (we also say "*e* stands for *v*", or "*e* denotes *v*"); in language L, the entity *e* is used to "make reference" to the entity *v*. In this context, *v* is usually said to be *referent* or *denotatum* of e. In Semantic Web *v* is said to be *value* of the reference *e*. The word "reference" treated as above can be regarded as a synonym for "label" - a word by which we also mean a sign with which we do not necessarily associate any meaning.

Semantic Web uses references which are written according certain standards, and we can consider them also labels. But Semantic Web allows for different interpretations of its references, allowing for a reference to potentially have many values at different interpretations and, thus, Semantic Web generalizes the notion of reference. We will not follow this terminological path, but use the practice of natural languages which have been most intelligence-consuming artefacts of humanity and which are more trustworthy as sources of correct terminology. Namely, we will treat references as just *proper* names, but allow also for *common* names. Moreover, we will not use "reference" interchangeably with "proper name", but treat reference as the relationship between names and their interpretations.

5. Names

By *name* I mean an entity to which we have attributed *sense* and which is used for reference. If we have attributed sense to a name we say that it has *sense* or, as it is more common to say, that it "makes sense". So, I consider sense to be an *attribute* of *name* – a property necessary for an entity to be called *name*. A *label* is used exactly to enable referencing the labelled entity, but as long as we did not consciously attribute certain meaning to the label, the label will not be considered a *name*.

A synonym for *sense* is *meaning*, but the word *sense* is more general and covers all phenomena related with a name, while *meaning* seems to cover only pure semantic phenomena. For example, we can say about certain actions of an agent that they "make sense" (*agent* is the term which denotes any entity capable of action). Also, it is hard to say that the grammar category of words said to be *particles* (like “of”, “to”, “from”, etc.) “have meaning”, but we say that they “make sense” or “have sense” due to their use in a special manner by an agent who speaks the language. Finally, something (an entity) can “make sense” to an agent and may not “make sense” to another agent, which shows that attribution of sense is essential – in the first case, the sense had been already attributed, and in the second case, it had not been attributed to the entity.

An expression of a natural language can be one word or a whole phrase, i.e. a name may be either simple (we will say “atomic”) or compound. Linguists denote the compound names by using square brackets. Say, they call [current president of Sierra-Leone]$_{NP}$ a noun-phrase and use it to make reference to the actual president of Sierra-Leone. Such a name might be even more convenient to use than the passport name if you don’t know the passport name of current president of Sierra Leone. Important to remember is that names can be compound and still treated conceptually the same as simple names – the composition, form or length of a name have nothing to do with the functions of a name: reference and sense.

While in conceptual analysis it is appropriate to use one word *name* for both simple and compound names, in metalingua, which we can use in practice, we will proceed like in linguistics and consider that the word *name* refer to only simple names and the word *expression* refer to compound names. Such a practice of using a word with one meaning in a language and another meaning in meta-language is common and acceptable.

Linguistics of a natural language uses a meta language for description of a natural language. Sometimes they inquire what is the antonym of “meta”. We will consider this to be *metro* (obtained by comparison with the Greek words *meta-polis*, “satellite city” and *metro-polis*, “main city”). Metalingua can be used as a meta language for any natural or formal language as metro-langauges.

An affirmative proposition is also a phrase – it is a phrase which “expresses a complete thought” as they teach us in elementary school. This allows us to treat propositions, which express truth, as names, which make sense. This, in turn, allows us, no matter how abstract this may sound, to consider truth is a partial case of sense (“truthful sense”) and reduce research of truth to the research of sense. Despite some amount of apparent artificiality, we will accept this abstraction, in order to be able to formulate a theory of sense by mapping the theory of truth (logic) into the theory of names related with metalingua which we will have to develop further. This is the methodology whereby the results of this paper have been obtained.

6. What is sense?

There is no widely accepted formalization of the notion of *sense* and I will introduce, as it is common in mathematics, an *extensional* definition of this notion (“extensional” means “in term of classes”). I will say the sense of a name ν, and denote it by **sense**(ν), to be the class of all possible values of a name. This definition conforms with the practice of saying that we know a name when we know what it denotes. Why sense is treated as a class? This is due to the ambiguity of reference and the class is meant to contain all the possible values to which a name, in this sense, can refer.

The name which refers to anything is natural to be *thing* and the OWL (Web Ontology Language, inversion intended) uses the reserved word **Thing** with same meaning. In the RDF (Resource Description Framework) language same role is played by the reserved word **Resource**. These names must be regarded as synonyms, i.e. names with the same sense, which as classes should coincide. So, **sense**(**Thing**) = **sense**(**Resource**). Also, the OWL has a reserved word **Nothing** for the name which has no values and we will use it here with the same meaning.

The two names **Thing** and **Nothing** are playing in semantics of metalingua the same role as the two Boolean values **True** and **False** play for the classical logic in set theoretic interpretation. The most important operation over names used in discourse is the operation of *modification* of one name *b* by another name *a* – an operation the result of which is denoted by ($a : b$). I define this operation in such a manner that it becomes an “image” in the “universe of sense” of the implication operation ($b \Rightarrow a$) from the “universe of logic”.

7. Conclusion

The ideas of this paper are in the process of development and alongside conceptual research they are also checked in computer software simulations at Semantic Soft, Inc. company, Republic of Moldova. In this paper, I have only outlined the ideas on how the precise semantics of metalingua can be developed. A complete account on this will come in future publications.

References

Drugus, I. 2007, 8A Wholebrain approach to the Web8, in Proceedings of the “Web Intelligence – Intelligent Agent Technology Conference8, Silicon Valley, pp 68-72.

Drugus, I. 2009a, 8Universics: an Approach to Knowledge based on Set theory8. in Knowledge Engineering Principles and Techniques, Selected Extended Papers (Cluj-Napoca, Romania, July 2-4, 2009), Presa Universitara Clujeana, pp. 193-200.

Drugus, I. 2009b, 8Universices – a Common Approach to Brain Informatics and Semantic Web8, in *Web Intelligence and Intelligent Agents*.
Frege, G. 1892, 8Uber Sinn und Bedeutung8, *Zeitschrift fur Philosophie und philosophische Kritik*, NF 100, S 25-50.
Russel, B. 1905, 8On Denoting8, *Mind*, New Series, vol. 14. No. 56. pp. 479-493