# The Applicability of Marginal Costs in Fixed Tangible Assets Analysis

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**Key words:** *marginal calculus, average value, marginal value, variation rate, efficiency.*
**Abstract:** Fixed tangible assets represent the highest proportion within the whole amount of fixed assets of an economic unit and significantly affect the profitability of its activity. These are an important source of information, mostly for the investors, for the creditors, commercial partners, for the state (tax collector unit) and even for the competitors and, generally, for the users of accounting information, but mostly, for the economic unit's own management, in order to make decisions regarding economic policies and investment.

Generally, optimizing economic activity implies two ways of action: either to minimize the costs involved in the activity, or to maximize the profits.

The application of marginal calculus in the economic field consists of economic phenomena analysis through mathematical modeling; this means to identify a function which can express the correlation between the cause – phenomenon and the effect – phenomenon, the evolution of the phenomenon being represented by curves.

Marginal calculus has applicability in the optimizing analysis of economic processes, in order to make the optimum choice regarding resources consumption or the net profit. For this purpose, one should examine an objective function and establish the effects of the independent variables' change on it, in order to identify those values of the variables, capable to optimize the function. It is important to remark that one should not take into consideration the sizes of the economic categories by themselves, but their variations, considered smaller and smaller. The applicability of marginal calculus in the analysis of tangible assets aims at the following: a) the improvement of the efficiencies of utilization; b) the improvement of the efficiency of the equipments costs (depreciation expenses).

The efficiency of the technical potential use can be expressed by means of the average and marginal indicators, each of them expressing a certain relationship between the production equipment and the activity volume.

Beside the improvement of the efficiencies of capital utilization (of the production factors, in general), another important goal for the economic unit is to minimize the relevant costs to a given level of production, as a condition to achieve a maximum profit.

The applicability of marginal calculus in managerial decision making has some limits, that is, it is possible to estimate only the associated effect with the unitary change of a variable. However, one should in fact be interested in the simultaneous effect of some variations.

In a competitive economy, the efficiency and the long-lasting characteristic of an economic unit depend on whether it has material potential, of which the functional and technical parameters correspond to the market economy's new requests and demands. This means that the technical potential of an economic unit must be adequate from the point of view of the volume, the structure and the quality, providing it the possibility to obtain a production corresponding to the exigencies of the internal and international market.

Fixed tangible assets represent the highest proportion within the whole amount of fixed assets of an economic unit and significantly affect the profitability of its activity. These are an important source of information, mostly for the investors, for the creditors, commercial partners, for the state (tax collector unit) and even for the competitors and, generally, for the users of accounting information, but mostly, for the economic unit's own management, in order to make decisions regarding economic politics and investment.

The specific nature of the fixed tangible assets consists in their capacity to remain in the economic unit for a period of time more than one year and to be the enterprise's technical support, according to their importance. Moreover, fixed tangible assets are not consumed on their first usage, but they gradually transmit their value to the goods and services to whose obtaining they contribute to; by their own nature, these assets are subject to a natural depreciation process. The cause of this depreciation consists not only in their usage during the current operating process, but also in the natural elements' action upon the environment they are part of; this leads to a *physical usage*, but fixed tangible assets are also subject to a *moral usage*, caused by the influence of the technical progress related to time.

One of the fundamental economic principles, that is the one according to which any activity must be profitable, justifies the application of the scientific methods in the real economic systems study, in the economic unit's activity analysis, in order to obtain some optimal solutions according to the specified objectives. Modern scientific proceedings concerning the solutions of the problems that an economic unit is confronted to, are based on mathematical methods and depend on the objective reality.

The application of marginal calculus in the economic field consists of economic phenomena analysis through mathematical modeling; this means to identify a function which can express the correlation between the cause – phenomenon and the effect – phenomenon, the evolution of the phenomenon being represented by curves.

Thus, suppose that between the analyzed phenomenon *(y)* and its determinative factors *(x)*, there is a relationship represented by the *y = f(x)* function; also, suppose that both the function and the curve which represents it are continuous and even in every point of the definition domain, then the function has a derivative and the curve has a tangent. In fact, the variations of the economic phenomena are not continuous, but discrete, being represented by histograms; however, one may consider that these variations are small enough to be represented by curves.

The fundamental concepts of the marginal analysis are the following: *average value, marginal value, variation rate* and *elasticity* of a function.

*The average value ( $\bar{y}$ )* represents the function variation *y = f(x)* on the interval values of *x*, designated with *[0; x]*.

$$\bar{y} = y / x = f(x) / x$$

The marginal value *($y_M$)* expresses the function variation "at the limit", corresponding to very small variations (with one unit) of the argument *x*, resulting from a given value.

$$y_M = \frac{\Delta y}{\Delta x} = \frac{y_1 - y_0}{x_1 - x_0}$$

In the case of *x* infinitesimal variations, marginal value becomes equal to function *y*'s derivative, in relation to the variable *x*.

$$y_M = \lim_{\Delta x \to 0} (\Delta y / \Delta x) = f'(x)$$

The variation rate *($R_{y/x}$)* of the function, in relation to the independent variable *x* reveals the percent to which the function *y* varies when the argument *x* varies with one unit. The calculation relation is:

$$R_{y/x} = \frac{\Delta y / y}{\Delta x} = \frac{\Delta ry}{\Delta x} = \frac{I_y - 100}{x_1 - x_0}$$, where

$\Delta ry(\%) = \frac{\Delta y}{y} = \frac{y_1 - y_0}{y_0} = \frac{y_1}{y_0} - 1 = I_y - 100(\%)$ = the relative variation of the function.

Another example of calculation of the variation rate is provided by the ratio between the marginal value of the function and the function itself *y*:

$$R_{y/x} = \frac{1}{y} \cdot \frac{\Delta y}{\Delta x} = \frac{y_M}{y}$$

*The elasticity* of a function in relation with its argument expresses the sensitiveness of the function to its argument variation. The calculation relations are:

$$E_{y/x} = \frac{\Delta y / y}{\Delta x / x} = \frac{\Delta ry}{\Delta rx} = \frac{I_y - 100}{I_x - 100}$$

$$E_{y/x} = \frac{\Delta y / y}{\Delta x / x} = \frac{y_M}{\bar{y}}$$

From the point of view of economic analysis, the elasticity is interpreted in terms of its *sign* and its *value*, related to the unit, as follows:

- the sign (±) shows the connection sense between *y* and *x*;
- the value related to the unit shows the elasticity (the sensitiveness) degree of the function with respect to its argument.

One may distinguish the following situations:

1. From the point of view of the *sign*:
   a) $E_{y/x} > 0$ expresses the evolution in the same sense of the phenomenon and factor (both of them increase or decrease);
   b) $E_{y/x} < 0$ represents the evolution in the opposite sense of the two variables (the increase of the argument determines the decrease of the function and vice versa).
2. From the point of view of the *value*:
   a) $E_{y/x} = 1$ expresses the equiproportional variation of the two variables ($I_y = I_x$, or $y_M = \bar{y}$);
   b) $E_{y/x} < 1$ represents the unproportional variation of the two variables ($I_y < I_x$, or $\Delta ry < \Delta rx$, or $y_M < \bar{y}$); the function is so – called relatively rigid in relation to argument *x*;
   c) $E_{y/x} > 1$ shows the unproportional variation of the two variables ($I_y > I_x$, or $\Delta ry > \Delta rx$, or $y_M > \bar{y}$); the function is relatively elastic in relation to argument *x*;
   d) $E_{y/x} = 0$ represents the maximum rigidity of the function y in relation to *x* ($\Delta ry = 0$, or $I_y = 100\ \%$, or $y_M = 0$, and $\bar{y} \neq 0$);
   e) $E_{y/x} \to \infty$ expresses an infinite elastic function ($\Delta rx = 0$, $I_x = 100\ \%$, and $\Delta ry \neq 0$).

If one takes into account the existing relations between one indicator (the function) and its determinative factors (the independent variables), the elasticity will be used in predictable analysis as leverage that will exert a specific effect on the indicator (on the function *y*) given certain predictable variations of the factors. Therefore, within economic and financial analysis, one should determine some degrees of leverage: Degree of Operating Leverage (defined as the percentage change in operating income that results from a given percentage change in sales), Degree of Financial Leverage (expresses the percentage change in earnings per share that results from a given percentage in earnings before interest and taxes), Degree of Combined Leverage (shows the impact of a change in sales or volume on bottom – line earnings per share).

Marginal calculus has applicability in the optimizing analysis of economic processes, meant to provide the optimum choice regarding resources consumption or the net profit. For this purpose, one should examine an objective function and establish the effects of the independent variables change on it, in order to identify those values of the variables, capable to optimize the function. It is important to remark that one should not take into consideration the sizes of the economic categories by themselves, but their variations, considered smaller and smaller.

The optimal value of a function $y = f(x)$ is reached when its marginal value is zero: $y_M = \frac{dy}{dx} = 0$. In order to see what the nature of this optimum (maximum or minimum) is, it is necessary to determine the second derivative's function sign. Thus, when $d^2y / dx^2 > 0$, function $y$ has a minimum value; in case of a negative second derivative ($d^2y / dx^2 < 0$), function $y$ has a maximum value. If $d^2y / dx^2 = 0$, then marginal value will reach an optimum, and the curve that represents the function goes above the turnaround point.

One may notice a significant relationship between average value and marginal value, that is average value reaches an optimum when its derivative is zero, that means the point where $y_M = \overline{y}$.

In a two variables function case: $y = f(x_1; x_2)$, establishing the optimum requires the calculation of partial derivatives, the differential of the function being zero: $dy = \frac{\partial y}{\partial x_1} \cdot dx_1 + \frac{\partial y}{\partial x_2} \cdot dx_2 = 0$, which means $y_M \cdot x_1 \cdot dx_1 + y_M \cdot x_2 \cdot dx_2 = 0$, that is $\frac{y_M x_1}{y_M x_2} = -\frac{dx_2}{dx_1}$.

The $-\frac{dx_2}{dx_1}$ ratio is called *marginal rate of substitution (MRS)* of the two variables and it expresses the degree to which a variable must increase, in order to compensate for the decrease of the other one, so that the value of the function may remain constant (the "–" sign shows the opposite evolution of the two variables).

$y$ function reaches its optimum when the marginal values reach zero: $y_M x_1 = 0$; $y_M x_2 = 0$. The nature of the optimum will be given by the increasing or the decreasing sense of the marginal values.

When there are restrictions between the variables of the function, the optimizing process of the objective function requires the usage of the Lagrange multipliers technique.

The applicability of marginal calculus in the analysis of tangible assets aims at the following: a) the improvement of the efficiencies of utilization; b) the improvement of the efficiency of the equipments costs (depreciation expenses).

a) The efficiency of the technical potential use can be expressed by means of the average and marginal indicators, each of them expressing a certain relationship between the production equipment and the activity volume.

*The overall efficiency of the equipment* represents the production (expressed physically or in monetary units) per unit of the used factor of production (which can be expressed by the number of equipments, or by the working period of time).

The calculation relation of the average product is as follows:

$$APK = \frac{Q}{K},$$

where: $APK$ – average product of fixed capital (annually, or per unit of time) (production equipment);

$Q$ – production value;

$K$ – used capital (expressed according to the number of equipments or according to their working time).

The difference between the real product of capital and the planned one (or the one of the preceding period) shows the efficiency of the equipment usage, and in the case of monetary expressed product, the change of the production's structure and prices becomes important (when prices are used to estimate production).

For an economic unit, the product's level expresses "a specific degree of operational coherence between the factors of production, the working methods and management" [3, 128]. The greater this coherence is, the better the product is obtained; furthermore the economic unit is capable of ensuring the continuity of the activity.

*The marginal product of capital (MPK)* is the additional output resulting from the use of an additional unit of capital:

$$r_M = \frac{\Delta Q}{\Delta K}$$

Unlike average product of capital, which expresses the usage of capital factor on its total amount, marginal product takes into account only the consequences of the last unit used of capital factor.

If $MPK > APK$, the latter will only increase over the contribution of the additional unit of the technical capital; however, when $MPK < APK$, the latter unit of the technical capital has a decreasing efficiency that acts in

the same sense on the average product, too. Thus, on a comparative point of view, the change of the average product of capital depends on its level and, at the same time, on marginal product value.

Between these two indicators, one may establish the following relations:

- *MPK* > *APK*, before *APK* reaches its maximum;
- *MPK* = *APK*, at the maximum point of the *APK*;
- *MPK* < *APK*, after the maximum of the *APK*.

Marginal product of capital, as a way of expressing the production equipment's efficiency, has some limits induced by the fact that it is a ratio between two absolute values, whose value is influenced by the measure unit used to express the amount of activity. In order to abstract this disadvantage, one introduced the concept of elasticity of the activity in relation to the change of consumed factors of production. The elasticity coefficient will be determined separately for each factor of production; henceforth, it is necessary to isolate the action of each factor, assuming, in turn, one variable, and the others, to be constant (in this case, capital factor is considered to be variable).

The calculation relation of elasticity coefficient is:

$$E_{Q/K} = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta K}{K}} = \frac{MPK}{APK}$$

The elasticity coefficient of production in relation to capital expresses the percentage growth rate of production, determined by 1 percentage growth of the consumed factor, all the other factors being considered constant. The values of the elasticity coefficient depend on the achievement indices of production ($I_Q$), in comparison to the adequate indices of the consumed factor quantity ($I_K$).

$$E_{Q/K} = \frac{I_Q - 100}{I_K - 100}$$

According to the specialty literature, the activity curve can be divided, in terms of elasticity's values, into three parts:

- first part, named *the increasing returns to scale* part, where $E_{Q/K} > 1$, if $I_Q > I_K$ and *MPK* > *APK*;
- the second part, named *the decreasing returns to scale* part, where $0 < E_{Q/K} < 1$, if $I_Q < I_K$ and *MPK* < *APK*;
- the third part, named *the negative returns to scale* part, where $E_{Q/K} < 0$.

b) Beside the improvement of the efficiencies of capital utilization (of the production factors, in general), another important goal for the economic unit is to minimize the relevant costs to a given level of production, as a condition to achieve a maximum profit.

Between the production value and the cost production, one may establish the following function: *C = f (Q)*. Similarly to the methodology of the production equipment product, marginal cost analysis requires, actually, a correlated analysis of the total, average and marginal costs.

*Marginal cost* represents the cost of the inputs necessary to produce an additional unit of output; *average cost* is the total cost divided by the produced number of goods (the output amount). In order to have decisional value, marginal cost analysis must be correlated with the average cost analysis and, eventually, referred to the product's price.

The relationship between total cost and the production volume can be expressed in terms of elasticity, as follows:

$$E_{TC/Q} = \frac{\frac{\Delta TC}{TC}}{\frac{\Delta Q}{Q}} = \frac{MC}{AC}, \text{ or } E_{TC/Q} = \frac{I_{TC} - 100}{I_Q - 100}$$

According to the specialty literature, the marginal cost curve can be divided into three parts, as follows:

- part I, of *increasing returns to scale*, where *MC* < *AC*, and *total cost is inelastic in relation to production* ($I_{TC} < I_Q$, *or* $E_{TC/Q} < 1$);
- part II, of *constant returns to scale (the optimal part)*, where *MC = AC = minimum*, and *total cost is equiproportional with production* ($I_{TC} = I_Q$, *or* $E_{TC/Q} = 1$);

- part III, of *decreasing returns to scale*, where *MC > AC*, and *total cost is elastic in relation to production ($I_{TC} > I_Q$, or $E_{TC/Q} > 1$).*

The marginal cost curve is more accentuated than the average cost curve, and the minimum of marginal cost is lower than the minimum of average cost. The differences between marginal cost and average cost can be justified by the existence and the characteristics of fixed costs (e.g. depreciation costs) which, on short term, remain constant as amount, and vary as a relative value. Average cost also includes fixed costs, while marginal cost represents only the variable costs, provided *the existing production capacities* exist. In case of production capacity extension (and consequently, of increasing fixed costs), marginal cost will also include a part of additional fixed costs required for the production achievement.

Marginal cost intersects the average cost curve on its minimum, which means that average cost is minimum at the point where it is equal to marginal cost.

In order to determine the efficiency of depreciation costs, one will consider the following formula:

$$I_D = \frac{D}{T} \cdot 1000 \text{ , where}$$

$I_D$ = index of depreciation costs at 1000 m.u. turnover;

D = depreciation costs;

T = turnover.

The increasing depreciation efficiency is the result of this indicator decrease and influences the costs' general efficiency growth. The turnover increase at a higher rate than the increase of depreciation determines the decrease of the indicator as a result of a sub – unitary elasticity: $E_{D/T} < 1$.

The depreciation value growth is the consequence of the investment, which determines the technical performances improvement of the fixed tangible assets, yet, only if the product of fixed tangible assets usage (which is being reflected in the turnover) increases faster than the depreciation value.

On a product lifetime one may identify a technical optimum and an economic one. *Technical optimum* corresponds to the production volume that allows the obtaining of a minimal average cost, which means that, from a technical and organizational point of view, one cannot obtain a cheaper production. *Economic optimum* corresponds to a volume of activity where the marginal cost is minimum. There are opinions according to which the economic optimum is reached when marginal cost is equal to the selling price.

The applicability of marginal calculus in managerial decision making has some limits, that is, it is possible to estimate only the associated effect with the unitary change of a variable. However, one should in fact be interested in the simultaneous effect of some variations.

*In conclusion,*

Generally, optimizing economic activity supposes two ways of action: either to minimize the costs implied by the activity, or to maximize the profits obtained from it. As a method of action, the way of maximizing the results acquired from the economic activity is not a practical one, because the maximization can be achieved by the growth of costs involved in obtaining the determined results. In the case of costs minimizing, costs that are effectively related to the production have great importance. These are provided by the general costs regarding the insurance of the conditions and spaces of production and by the specific costs of every equipment involved in the economic process. *General costs* are equally divided between the existing equipment, or at the entity's level where the production is obtained (one or more economic units). *Specific costs* are generated by the raw materials and consumables and utilities consumption, in order to carry on the production processes; these costs are influenced by the technology used.

The analysis of fixed tangible assets' volume of an economic unit can not be reduced only to their value. Due to their nature, these assets are subject to natural depreciation processes. The cause of this depreciation consists not only in their usage in the current operating process, but also in the natural elements' action of the environment they are part of; this leads to a *physical usage*, but fixed tangible assets are also subject to a *moral usage*, caused by the influence of the technical progress related to time.

The analysis of fixed tangible assets, with all it implies – volume, structure, status, quality, dynamics etc. –, represents an internal concern for the economic unit, this way, one may determine the way investment had materialized and, at the same time, the correlation with the produced effects' dynamics. Hence there appears the economic unit's interest regarding financial capacity, referring to competition and to penetrating the internal and the external market of its goods or services.

Placing maximum use on technical and productive potential of an economic entity requires the methods and the decisions optimizing techniques, the instruments and the proceedings usage, necessary for the continuous improvement of the whole economic activity quality. Through their efficiency, these optimizing methods and techniques are the best way to activity improvement, and also a way of emphasizing the internal resources of the economic unit.

Of great importance is the use of computers in order to solve mathematical optimizing models (for example the optimization of the equipment position within production sections, and of the sections within the enterprise, based on certain software, the optimization of the production stocks dimension, the optimization of the planning and launching orders on condition of a complete charging equipment etc.).

Economic unit's technical potential optimization methods and techniques ensure the economic efficiency improvement, which is the basic condition for economic activity development.

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