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# *Mathematical Modeling of a Production Enterprise*

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***Abstract:*** *This article presents an attempt of modeling and mathematical simulation of a manufacturing enterprise with the help of mathematical systems theory and the theory of finite automata. It also proposes a special type of enterprise-type modularized production which corresponds very well to this type of mathematical modeling. For math theory there are used concepts of systems and simulation is done using finite automata.*
***Keywords:*** *finite automata, enterprise modeling, mathematical systems theory, mathematical modeling, simulation, manufacturing*

## Introduction

In general a model (M) of an object (system) real S, is a remote system, denoted (S') which in some aspects is equivalent with the original system (S), denoted (S) ~ (S'). From the determination of relationships on (S') can be derived easily the corresponding relationships on (S). Equivalent of (S) with (S ') may be exact or approximate. Thus, in the formal theories can build systems (S') which are rigorously equivalent real system (S) [3, 82]. In other cases the model is a theoretical construction of the approximate part of the reality. In this case the model is an approximation of the reality. If this theoretical construction of the model (S') is given by mathematical relations, then these mathematical relationships together with their interpretation are a mathematical model of the real object (S). In science the concept of model is used for the purpose of simplified representation of a real system. This representation is a representation of its internal composition (the internal structure of the model) as well as its operation (function model) [2, 106]. Modeling is a construction on real objects and research of models of the real objects [2, 106].It is also possible to use previously constructed models to create other models. In this paper we use two mathematical theories (mathematical model themselves) to model the structure and operation of manufacturing enterprises [1, 175]. These two mathematical theories are mathematical system theory and graph theory.

## 1. Definitions of Mathematical Systems

Definition 1. According to the paper [4, 3] system is a mathematical structure of the form:

$$S = (T; X; U; V; Y; \gamma ; \eta)$$

where we have:

a) the set T is a nonempty set called during system S, the interpretation of "time" ($T \neq \Phi$). Here the set T modeling the time of is investigation (and evolving) the real system (which are modeling by the mathematical structure S). If $T = \mathbf{N}$ then we says that S is discrete-time system, if $T = T_1 \subset \mathbf{N}$ and $T_1$ is a finite set ($T_1 = \{1,2,3,...,n\}$, $n \in \mathbf{N}$), then it is said that the system is discrete and finite time (that is, the real system which it modeling by the mathematical structure S has a rise time equal to 1 and a dead time equal to $n \in \mathbf{N}$). So the whole „life" of the system S runs between times 1 and n, $n \in \mathbf{N}$. Presence the set S on the T system allows the system to be simulated in order to anticipate its future development. In this case the system S is called dynamic system.

b) the set X is called the set input system elements S. The elements of the X is input in S at each time point $i \in T$, denote by $x_i$. So we have $X = \{x_1,x_2, ... ,x_n, ...\}$ if $T = \mathbf{N}$, and $X = \{ x_1,x_2, ... ,x_n\}$, if $T = \{1,2,3,...,n\}$. The elements of the set X can be any kind of mathematical entities (numbers, vectors, functions, matrices, sets of numbers etc., or other systems ($S_i$), or more complex mathematical structures, depending upon the complexity of the real object that it modulates by the S above).

c) U is called the set of the entry segments into the system S. The elements of U are graphics (like the algebraic set, but is not algebraic geometric figures) of some functions of the form: u: $[t_1, t_2] \to X$, with $t_1$, $t_2$ from T and $t_1 \le t_2$, functions called input segments in system S. So this input segment is denoted as $u([t_1,t_2])$ and have the form $u([t_1 ,t_2]) = \{(i,u(i)) / i \in \{t_1 ,t_2]) \}$. The functions of the form u: $[t_1 ,t_2] \to X$, with $t_1$, $t_2$ from T and $t_1 \le t_2$ , they are from of the giving infinite set of the functions, the set of the form: $F = \{u_1,u_2, ... ,u_n, ...\}$, if T = **N**, and she is the form $F = \{u_1,u_2, ... ,u_n\}$, if T = {1,2,3,...,n}. In both cases, the functions from F are of the general form: $u_i$: $[t^i_1 ,t^i_2] \to X$, $i \in \mathbf{N}$ and have the following significance: inputs in the S system between time $t^i_1$ and $t^i_2$ are given by the function $u_i$, i.e. for any k so that the $t^i_1 \le k \le t^i_2$, we have $x_k = u_i(k)$. It is found that the $u_i$ functions in the set of functions *F*, do not have the same domain of definition (in general). So with these functions can not be make (in generally speaking) adding or subtracting operations, but only multiplication operation with real numbers (if $x_k = u_i(k)$ allow such operations). In general these functions give us the form of evolution within a certain time interval of the S system entries (this if the entries in the S system also depend on the time, $i \in \mathbf{N}$). So the set of the input segments for the S system is: $U = \{u_i([t^i_1 ,t^i_2]) / t^i_1 \le t^i_2,\ t^i_1, t^i_2 \in \mathbf{N},\ i \in \mathbf{N}\}$, i.e. a lot of input segments in S.

d) the set V is called the set of the internal states of the S. This set can be finite or not. If V is the finite set, then the S system is called system with finite set of states, and if V is infinite set then the S system is called system with infinite set of states. We'll note on V on such way: $V = \{s_1,s_2, ... ,s_n,. ...\}$ if V are the infinite set, and with $V = \{s_1,s_2, ... ,s_n\}$ if V are the finite set. The elements of V are called the internal states of the system S, and they modeling the internal structure of the real system (which are modeling by the S). Among the states of S, one has a particular significance, namely the condition of $s_0$ called the initial state of the system S. From this state begins to evolve the system. The internal structure of the S system contains all the "history" of its evolution over time (past present and future). These internal states and (from the time $i \in T$) of S, together with the inputs $x_i$ (from the respective moments of times), uniquely determine the replies of the S system (its outputs).

e) the set Y is called the set of outputs of the system S(or its replies). In general, the set Y is a lot of elements, finite or not, and we write it as follows:

a. $Y = \{y_1,y_2, ... ,y_n,. ...\}$.

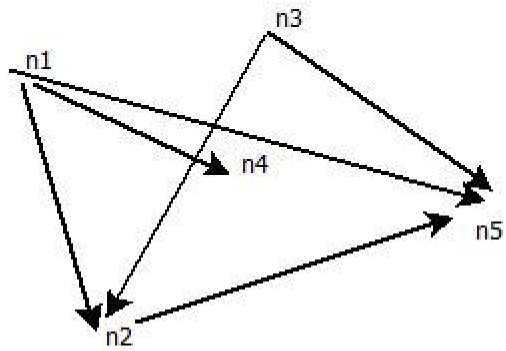
f) the function $\gamma : X \times V \to Y$ is called the response (or output) function of the system S to entries $x_i$ at the time $i \in T$. It models how the system responds if at the time $i \in T$ receive an entry $x_i$. So whatever $i \in T$, we have $y_i = \gamma(x_i,s_i)$.

g) the function $\eta : X \times V \to V$ is called the transitional function of the internal states of the S system. This function makes the transition of the system S from a $s_i$ (from the moment $i \in T$) in another state, denoted by $s_{i+1}$ (at the next time $(i+1) \in T$). This passage depends (through the function $\eta$) of the condition $s_i$ at the time $i \in T$, and she depends of the input $x_i$ at the time $i \in T$. So we have the follow relation: $s_{i+1} = \eta( x_i , s_i)$ in every time $i \in T$.

This is the mathematical model of a real system, its behavior in time being given by knowledge at any time $k \in T$ of the configuration of its inputs (i.e. the input segments $u_i$: $[t^i_1 ,t^i_2] \to X$, $i \in \mathbf{N}$ such that $t^i_1 \le k \le t^i_2$ , $t^i_1 ,t^i_2 \in T$), and his answers $y_k$ (from the same moment of time $k \in T$ ). Knowing the time behavior of such a system shows how the real system S will evolve over time.

In order to be able to analyze the evolution over time of such a system, it is naturally and uniquely attached to a graph $G_S = (N,A)$, called the evolution graph of the S system S [6, 133], in this way: $G_S$ graph' nodes are the S-system states i.e. elements of the form $s_i$, $i \in T$, so N=V, and the arches of the graph $G_S$ are the form $(s_i, s_k)$ if there is a moment of time $k \in T$ such that $s_j = \eta( x_k, , s_k)$, i.e. if there is in the evolution of S a moment of time $k \in T$, such that from the state of $s_k$ at that time and with the $x_k$ entry at that time and with the function $\eta$ of transitional states to get in the state $s_i$. This graph allows for simpler viewing of the time evolution of the S system.

Definition 2. [5, 10] It's called a graph, a mathematical structure of the form: G = (N, A) where the elements N and A are called in such way: N = the set of the G-graph' nodes, and A = the set of the G-arches. The set N are the form: $N = \{n_1,n_2, \dots ,n_k, \dots\}$, witch the nodes are; $n_1,n_2, \dots ,n_k, \dots$, and the set A are the form $A = \{( n_1,n_2), (n_3,n_5), \dots , (n_i,n_j), \dots\} \subset N \times N$, so A is a nonempty subset of pairs of elements in N. In the figure below we have an example of such a graph in which: $N = \{n_1,n_2,n_3,n_4,n_5\}$ is the set of G's nodes, and the set of his arches is: $A = \{(n_1,n_2),(n_1,n_4),(n_1,n_5),(n_2,n_5),(n_3,n_2),(n_3,n_5)\}$.

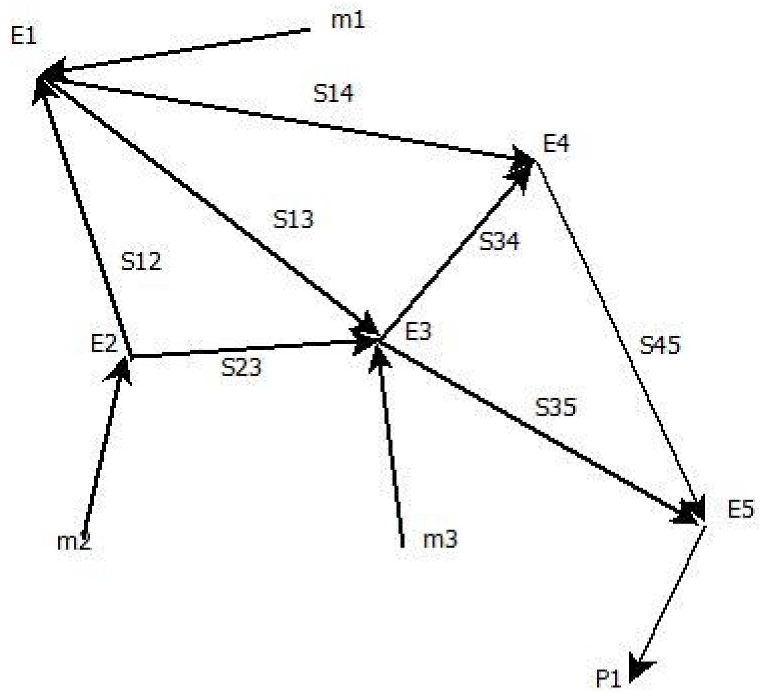



At such graphs we can have labels on the arches.

With the help of these mathematical instruments we will build in the following a model S of a production enterprise, denoted (PU) that has a certain particularity (which can represent the future of these types of enterprises) [5, 52].

## 2. The Main Result

We still assume that the enterprise (PU) has a number of production elements, modularized (sections, machines, installations etc.), denoted by: E1, E2,..., En. It is also assumed that these elements can be mounted together in different configurations of the form of graphs (which are manufacturing lines) as in the following figure:




In this example we have a graph that represents a combination of $E_1$, $E_2$, $E_3$, $E_4$ and $E_5$ elements that constitute a manufacturing line of the final product noted with $P_1$. Elements $E_1$, $E_2$, and $E_3$ have as inputs initial materials: $m_1$, $m_2$, and $m_3$, and the arches of this graph are labeled with: $S_{14}$, $S_{13}$, $S_{12}$, $S_{23}$,

$S_{35}$, $S_{45}$and $S_{34}$ and they represent the sub-assemblies constructed by the elements from which those arches depart and which are used further in the elements of the end's of arches. For example, the $S_{14}$ sub-assemblies are manufactured by the $E_1$ element of the $m_1$ initial material and are intended for the $E_4$ element. Likewise, in $E_3$ enters the sub-assemblies $S_{23}$, $S_{13}$ (supplied by $E_2$ and $E_1$) and the initial material $m_3$ and will produce the $S_{34}$ and $S_{35}$ subassemblies. In this way a certain $P_k$ product can be constructed from several combinations of elements of type $E_i$ and with different initial materials of type $m_j$. For a specific $P_k$ product we will note with $\mathcal{G}_k$ the set of all configurations (which are actually graphs with nodes in elements of type $E_i$ or $m_j$ and arches are labeled with the subassemblies of type $S_{ij}$) that can generate by manufacturing the $P_k$ element. So the set $\mathcal{G}_k$ is the form:

$\mathcal{G}_k = \{G_1(P_k), G_2(P_k), \ldots, G_m(P_k)\}$in which $Gi(P_k)$ is a graph (manufacturing line configuration) that gives the $P_k$ product a production.

If at a time $i \in T$is manufactured the product $P_i$, and if at the next time $(i + 1)$ is received the command to manufacture another product $P_{i+1}$, then from the set of configurations $G_{(i+1)}$ will choose that configuration $G_j(P_{i+1})$ that will build on $P_{i+1}$.

We are now in the measure of modeling such enterprises through the systems of the form of the introduction.

Let be the system form: $S = (T;X;U;V;Y;\gamma;\eta)$ in which we have:

a) T is discreet and infinite time, so $T = \mathbf{N}$, $T = \{0.1, \ldots, n, \ldots\}$,

b) X is the discreet set of inputs, $X = \{x_1, x_2, \ldots, x_n\}$ in which the elements $x_i$ have the following significance: $x_i = (K(P_i), m_i)$ with $m_i$= are the initial materials entering the manufacturing for a scheme (stare) $G_j(P_i)$ of $G_k$, and $K(P_i)$ are the technical specifications of the future product $P_i$ (to be constructed with the $G_j(P_i)$ manufacturing line chosen from $G_k$). These $K(P_i)$ elements are set of "words" from a certain "formal language" L that "understands" by the S system.

c) U are the set of the input segments of the form:
$U = \{u_i([t^i_1, t^i_2]) / t^i_1 \le t^i_2,\ t^i_1, t^i_2 \in \mathbf{N},\ i \in \mathbf{N}\}$ here the set of the functions $\mathcal{F}=\{u_i / i \in \mathbf{N}\}$ are given.

d) V is the set of S's states, which in this case consists of an element of the set of the forms of $G_k$, $k \in \mathbf{N}$ So we have: $V = \{s_i = G_j(P_i) / G_j(P_i) \in \mathcal{G}_i\}$. As a generalized case of the set V of paragraph 1. We can consider that the elements of V are elements of the form $G_k$, $k \in \mathbf{N}$, i.e. to have the set $V = \{\mathcal{G}_k /\ k \in \mathbf{N}\}$. So the states of the S in this case are even the $G_k$ sets. These states of S we will call generalized states of the S system (because the usual states are elements of $G_k$).

e) Y is the set of the output's of the S system , which are in this case $Y = \{y_1, y_2, \ldots, y_n, \ldots\}$ in which $y_n = P_n$, $n \in \mathbf{N}$,

f) the function: $\gamma : X \times V \rightarrow Y$ (the system output's function or response function of the system S) is in this case $y_n = P_n = \gamma(K(P_n), \mathcal{G}_n)$ or sau $y_n = P_n = \gamma(K(P_n), G_j(P_n))$ and $G_j(P_n) \in \mathcal{G}_n$, $n \in \mathbf{N}$, in which $x_n = K(P_n)$, and $s_n = \mathcal{G}_n$ or $s_n = G_j(P_n))$ with $G_j(P_n) \in \mathcal{G}_n$, $n \in \mathbf{N}$.

g) the function: $\eta : X \times V \rightarrow V$, called the transition function of the internal states of the S system, is in this case the function that reconfigures the assembly lines at the moment, $i \in \mathbf{N}$, denoted by $s_i = G_j(P_i)$ with $G_j(P_i) \in \mathcal{G}_i$, $i \in \mathbf{N}$, in another assembly line with $s_{i+1} = G_j(P_{i+1}))$ with $G_j(P_{i+1}) \in \mathcal{G}_{i+1}$, $(i+1) \in \mathbf{N}$, this passage is also made according to $x_i = K(P_i)$. So we have: $s_{(i+1)} = \eta(K(P_i), G_j(P_i))$, and $G_j(P_i) \in \mathcal{G}_i$, $i \in \mathbf{N}$.

## Conclusion

This model can now be simulated (with the help of the associated graph), and thus we can analyze the behavior of this system in the future.

Using this unit model with modularized elements of production, it can be predicted (by simulation) the future behavior of the system.

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